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Modeling of the Dependence Between the Space-Time Processes

1. Introduction

The main thesis of the paper is a statement, that the basis of an appropriate modeling of the dependence between the space-time processes is to consider their internal structures.

The space-time processes are characterized by the sets of double indexed variables $X_{u,t}$, the so-called random fields. The models of such random fields, discussed in sections 2 and 3 of the paper, have got an essential significance for the specification of the dependence between the space-time processes.

The models, discussed in section 4, take into account the principle of the time, space and space-time „dynamics” – which manifests itself by the specification of the appropriate time lags, space shifts, and also simultaneous space-time shifts in the modeling of the dependence – and the principle of congruency as well, which is an extension of the principle of the congruency, used in econometrics for the linear dynamic modeling of the dependence of the stochastic processes.

The advantages for the modeling of the dependence of economic space-time processes resulting from such an approach are pointed out. The theoretical considerations are illustrated – in section 5 – by an empirical example, which refers to the dependence between unemployment rate and average monthly gross wages and salaries in enterprise sector in Poland. In section 6 the conclusions are formulated and the directions for further investigations are announced.

2. Modeling of the Trend-Seasonal Structure

Heterogeneous/non-stationary, with regard to the mean values, space-time economic processes may be modeled using the polynomial space-time trend functions and the seasonal component models.

Let $X_{\mathbf{u}_i, t}$ denote the space-time process, observed in the spatial units i , with the co-ordinates of the location $\mathbf{u}_i = (u_{1i}, u_{2i})$, in the time t . The expression of the form:

$$f(u_{1i}, u_{2i}; t) = \sum_{r_1 + r_2 + r_3 \leq r} \gamma_{r_1, r_2, r_3} u_{1i}^{r_1} u_{2i}^{r_2} t^{r_3} \quad (1)$$

presents the spatio-temporal trend of degree r .

The model of the space-time process with the trend and seasonality takes the form:

$$X_{\mathbf{u}_i, t} = \sum_{r_1 + r_2 + r_3 \leq r} \gamma_{r_1, r_2, r_3} u_{1i}^{r_1} u_{2i}^{r_2} t^{r_3} + \sum_{k=1}^m d_k Q_{kt} + \eta_{\mathbf{u}_i, t}, \quad (2)$$

where: Q_{kt} – seasonal dummies,

$\eta_{\mathbf{u}_i, t}$ – homogeneous/stationary space-time residual process.

3. Modeling of the Autoregressive Structure

The construction of the autoregressive space-time models is based on the statement, that the values of a phenomenon observed at the established points in time and space are dependent on the previous observations of the phenomenon at the other points in space. Connections among the variables in different units in space depend in a systematical way on the spatial distance, likewise the dependence in time depends on the time distance. The dependence among the neighbours of different orders is considered.

For expressing the connections of the observations of the variable in one place with the observations of the same variable in other places, it is conveniently to refer to the idea of the spatial lag, which in practice is named the spatial shift operator.

The spatial shift operator differs from the temporal shift operator, because the last one causes shifts of the variable by one or more periods backwards, whereas the spatial operator acts into different directions, with regard to the fact, that the direction of the shifts in space may be various.

The definition of the spatial shift operator depends on the spatial data arrangement and on what is known in advance about the investigated phenomenon, e.g. whether there is well founded the assumption, that the influence of the variable located in the given place on such the variable located in another place

depends mainly on the distance between the locations and does not depend on the direction or whether it should be assumed to depend on the direction as well¹.

The starting point to specify the spatial shift operator is the identification of the neighbours of each place i on the lattice² D , with regard to the well-defined criterion of specification (i.e. a common border for the so-called nearest neighbours). The neighbours of the first, second etc. order are identified. The appropriate sets of the neighbours are denoted by: $N_1(i)$, $N_2(i)$, ... (generally, $N_s(i)$, where s denotes the order of the neighbourhood).

When the sets of neighbours are fixed for each place i , the spatial shift operator of the order s (i.e. $L^{(s)}$) may be defined as follows:

$$L^{(s)} X_{i,t} = \sum_{j \in N_s(i)} w_{ij}^{(s)} X_{j,t}. \quad (3)$$

From the considerations above it appears, that the spatial shift operator is the operator of the lags distributed in space rather than the shift operator in the given direction³.

It is assumed, that the weights in (3) satisfy the following conditions:

- 1) $w_{ij}^{(s)} \geq 0$,
- 2) $w_{ii}^{(s)} = 0$,
- 3) $\sum_{j \in N_s(i)} w_{ij}^{(s)} = 1$.

Usually, the spatial weights are established *a priori* by the researcher. They may reflect the length of common borders, number of roads, railways, geographical or economic distance between the regions.

Using the concept of the spatial operator $L^{(s)}$ the autoregressive space-time model of order l in space and q in time, marked by STAR(l, q), i.e.:

$$X_{i,t} = \sum_{s=0}^l \sum_{\tau=1}^q \alpha_{s\tau} L^{(s)} X_{i,t-\tau} + \varepsilon_{i,t}. \quad (4)$$

may be defined.

In model (4) the so-called „pure” spatial autodependence, i.e. the dependence among spatial units in the same time is not considered. Generally it is justified. Since it is possible to agree with the argumentation, that the events in dif-

¹ Sometimes the spatial shift operator is presented as the so-called structure of the spatial shifts in a model. Some different spatial shifts structures are possible. On their importance for defining the so-called STARMA models (space-time autoregressive-moving-average models) and for the properties of the models, see e.g. Hopper and Hewings (1981).

² The definition of the spatial lattice may be find in Cressie (1993).

³ See, e.g. Giacomini, Granger (2004).

ferent points in space do not influence the events in other locations at once, because the realization of the results of the influence needs some time lag. However, an attention should be paid, that the assumption of the lack of instantaneous spatial dependence is important, provided the time distance between the observations is smaller than the real time lag of reaction. If the mechanism generating the course of the phenomenon creates it with frequency greater than the frequency with which the data are observed, then spuriously instantaneous influences may appear. Thus, the problem whether the space-time autoregressive model should include the clear spatial component depends on the scale of time realization and measurement of the phenomenon. Furthermore, while the instantaneous causal dependence may be doubtful, the spatial correlation in the same time (the so-called spatial autocorrelation) is obviously possible.

In the paper the considerations are limited to the space-time autoregressive models of the form (4). Thus, it is assumed that the spatial dependence requires at least one time lag before its effects are occurred.

4. Modeling of the Dependence Between Processes

The important idea of the modeling of the dependence between space-time processes, taking into account the structure of the connections in time and space is the congruent modeling of random fields. The concept proceeds with the econometric congruent modeling, which refers to the stochastic processes.

The author of the paper has already undertaken some attempts to construct the congruent models for random fields and to investigate their properties on the ground of the theoretical considerations and on the basis of the generated data⁴ as well. In the paper the empirical example of the modeling of the dependence of two space-time processes is presented⁵.

In this case the procedure of the congruent model construction is following :

- 1) The models with spatio-temporal trend and seasonality are identified:

$$X_{\mathbf{u}_i,t} = \sum_{r_1+r_2+r_3 \leq r^{(x)}} \gamma_{r_1,r_2,r_3} u_{1i}^{r_1} u_{2i}^{r_2} t^{r_3} + \sum_{k=1}^m d_k^{(x)} Q_{kt} + \eta_{\mathbf{u}_i,t}^{(x)}, \quad (5)$$

$$Y_{\mathbf{u}_i,t} = \sum_{r_1+r_2+r_3 \leq r^{(y)}} \delta_{r_1,r_2,r_3} u_{1i}^{r_1} u_{2i}^{r_2} t^{r_3} + \sum_{k=1}^m d_k^{(y)} Q_{kt} + \eta_{\mathbf{u}_i,t}^{(y)}. \quad (6)$$

- 2) The space-time processes $\eta_{\mathbf{u}_i,t}^{(x)}$, $\eta_{\mathbf{u}_i,t}^{(y)}$ are identified as autoregressive ones, and modeled as:

⁴ See Szulc (1998, 2003).

⁵ See also Szulc (2007).

$$\eta_{\mathbf{u}_i,t}^{(x)} = \sum_{s=0}^l \sum_{\tau=1}^q \alpha_{s,\tau} L^{(s)} \eta_{\mathbf{u}_i,t-\tau}^{(x)} + \varepsilon_{\mathbf{u}_i,t}^{(x)}, \quad (7)$$

$$\eta_{\mathbf{u}_i,t}^{(y)} = \sum_{s=0}^h \sum_{\tau=1}^p \beta_{s,\tau} L^{(s)} \eta_{\mathbf{u}_i,t-\tau}^{(y)} + \varepsilon_{\mathbf{u}_i,t}^{(y)}. \quad (8)$$

- 3) The equation of the dependence for the white noise space-time processes $\varepsilon_{\mathbf{u}_i,t}^{(x)}$, $\varepsilon_{\mathbf{u}_i,t}^{(y)}$ is constructed:

$$\varepsilon_{\mathbf{u}_i,t}^{(y)} = \rho \varepsilon_{\mathbf{u}_i,t}^{(x)} + \varepsilon_{\mathbf{u}_i,t}, \quad (9)$$

where: $\varepsilon_{\mathbf{u}_i,t}$ – white noise independent of $\varepsilon_{\mathbf{u}_i,t}^{(x)}$.

- 4) The congruent model for real processes is obtained by taking the residual processes from (5) and (6) and by substituting them into (7) and (8) respectively and finally, the transformed (7) and (8) – into (9). As the result the following model is obtained:

$$\begin{aligned} Y_{\mathbf{u}_i,t} = & \sum_{r_1+r_2+r_3 \leq r} \theta_{r_1,r_2,r_3} u_{1i}^{r_1} u_{2i}^{r_2} t^{r_3} + \sum_{k=1}^m d_k Q_{kt} + \rho X_{\mathbf{u}_i,t} \\ & + \sum_{s=0}^h \sum_{\tau=1}^p \beta_{s,\tau} L^{(s)} Y_{\mathbf{u}_i,t-\tau} + \sum_{s=0}^l \sum_{\tau=1}^q \alpha_{s,\tau}^* L^{(s)} X_{\mathbf{u}_i,t-\tau} + \varepsilon_{\mathbf{u}_i,t}, \end{aligned} \quad (10)$$

where: $r = \max\{r^{(x)}, r^{(y)}\}$, $\alpha_{s,\tau}^* = -\rho \alpha_{s,\tau}$.

Taking into account the trend-seasonal component in the model (10) the heterogeneous/non-stationary mean value is removed from the processes $X_{\mathbf{u}_i,t}$, $Y_{\mathbf{u}_i,t}$, therefore the parameters: $\alpha_{s,\tau}^*$, $\beta_{s,\tau}$, ρ measure the dependence between homogeneous/stationary components of these processes. Apart from the current dependence between the processes $X_{\mathbf{u}_i,t}$, $Y_{\mathbf{u}_i,t}$, measured by the parameter ρ , in the model (10) the dependence which is lagged in time and space is taken into consideration. The influence of the explanatory phenomenon – observed at the same points in time and space at which the explained phenomenon is observed – is separated from the influence of the phenomenon, observed somewhere else and some other time. These influences are measured by ρ and $\alpha_{s,\tau}^*$ respectively. The parameters $\beta_{s,\tau}$ reflect the connections in time among the magnitudes of the explained phenomenon, observed in the neighbouring spatial units. Thanks to *explicite* separation of the variables $L^{(s)} X_{\mathbf{u}_i,t-\tau}$, they will not contain the so-called indirect influences on $Y_{\mathbf{u}_i,t}$.

Specification of the model (10) results from the investigation of the internal structure of individual processes. It is the initial model, which after estimation of the parameters requires the insignificant components to be reduced.

5. Empirical Example

The empirical example refers to the dependence between unemployment rate and average monthly gross wages and salaries (in PLN) in the enterprise sector in Poland by voivodships in the period: January 1999 – December 2006. The data are from *Statistical Bulletins of voivodships* from the appropriate periods and from the internet sources: <http://www.stat.gov.pl>. The statistic sample consists of two data sets, including every 96 time observations for each of 16 spatial units, i.e. 1536 observations together.

The collected data as well on unemployment as on wages and salaries demonstrate trend and seasonal changes. The suggestion that the data may be spatial correlated is reasonable as well.

A. Investigating the trend and seasonality

The models of polynomial functions of the spatio-temporal trend with seasonality were considered. For wages and salaries the model of the form⁶

$$\begin{aligned}
 x_{i,j,t} = & 884.412 + 504.143i + 700.863j + 3.54733t \\
 & - 504.188i^2 - 598.971j^2 - 0.08795t^2 + 568.350ij \\
 & + 0.10905it + 0.98416jt - 78.7202i^2j + 0.00053i^2t \\
 & - 40.6421ij^2 + 0.003877it^2 - 0.173863ijt + 90.7726i^3 \\
 & + 95.2109j^3 + 0.0007t^3 - 0.05476j^2t - 0.0049375jt^2 \\
 & - 62.1517Q_{1t}^* - 65.4650Q_{2t}^* + 6.82298Q_{3t}^* - 7.14584Q_{4t}^* \\
 & - 43.6428Q_{5t}^* - 12.3288Q_{6t}^* + 4.9931Q_{7t}^* - 3.51258Q_{8t}^* \\
 & - 4.9217Q_{9t}^* - 10.9073Q_{10t}^* + 46.1711Q_{11t}^* + u_{i,j,t}^{(x)}
 \end{aligned} \tag{11}$$

was chosen.

The model (11) presents the spatio-temporal trend of the 3rd degree and seasonality. The most of the parameters of the model are significant. The model fit in this case is not high ($R^2=0.456623$).

⁶ For the further notation to be simplified, the index u_i was changed by (i, j) .

The analysis of the trend and seasonal changes in the space-time process of unemployment allowed to fit the model (12). The most of its parameters are significant. The coefficient R^2 for the model equals 0.828638.

$$\begin{aligned}
 y_{i,j,t} = & 39.4207 - 9.3825i - 32.7187j + 0.485652t + 6.48565i^2 \\
 & - 18.3333j^2 + 0.001957t^2 - 6.44723ij - 0.0452969it \\
 & - 0.091844jt + 0.349141i^2j + 0.00043i^2t + 1.01326ij^2 \\
 & + 0.000312it^2 + 0.00435ijt - 0.956293i^3 - 2.79388j^3 \\
 & - 0.000002t^3 - 0.00153j^2t - 0.0007jt^2 + 0.82513Q_{1t}^* \\
 & + 0.895193Q_{2t}^* + 0.75089Q_{3t}^* + 0.316561Q_{4t}^* - 0.1647Q_{5t}^* \\
 & - 0.334965Q_{6t}^* - 0.364415Q_{7t}^* - 0.443557Q_{8t}^* \\
 & - 0.497265Q_{9t}^* - 0.610573Q_{10t}^* - 0.419296Q_{11t}^* + u_{i,j,t}^{(y)}.
 \end{aligned} \tag{12}$$

B. Investigating the autoregressive structure

The purpose of the analysis of the autoregressive structure of the investigated processes was to identify significance of the largest time lags and spatial shifts. For both the processes the time lags of the 12th order are significant, whereas the spatial shifts are significant only of the 1st order. Thus STAR(12, 1) models were estimated. E.g. for wages and salaries the model took the form:

$$\begin{aligned}
 u_{i,j,t}^{(x)} = & 0.353579 + 0.118691u_{i,j,t-1}^{(x)} + 0.08124u_{i,j,t-2}^{(x)} + 0.00984u_{i,j,t-3}^{(x)} \\
 & + 0.02756u_{i,j,t-4}^{(x)} + 0.046427u_{i,j,t-5}^{(x)} + 0.01471u_{i,j,t-6}^{(x)} + 0.0379u_{i,j,t-7}^{(x)} \\
 & - 0.02156u_{i,j,t-8}^{(x)} - 0.03754u_{i,j,t-9}^{(x)} + 0.0206u_{i,j,t-10}^{(x)} + 0.07003u_{i,j,t-11}^{(x)} \\
 & + 0.62846u_{i,j,t-12}^{(x)} + 0.16958L^{(1)}u_{i,j,t-1}^{(x)} + 0.03398L^{(1)}u_{i,j,t-2}^{(x)} \\
 & + 0.0183L^{(1)}u_{i,j,t-3}^{(x)} - 0.00355L^{(1)}u_{i,j,t-4}^{(x)} + 0.065L^{(1)}u_{i,j,t-5}^{(x)} \\
 & - 0.01289L^{(1)}u_{i,j,t-6}^{(x)} - 0.0699L^{(1)}u_{i,j,t-7}^{(x)} + 0.0003L^{(1)}u_{i,j,t-8}^{(x)} \\
 & + 0.02965L^{(1)}u_{i,j,t-9}^{(x)} - 0.0565L^{(1)}u_{i,j,t-10}^{(x)} + 0.0203L^{(1)}u_{i,j,t-11}^{(x)} \\
 & - 0.28162L^{(1)}u_{i,j,t-12}^{(x)} + e_{i,j,t}^{(x)}.
 \end{aligned} \tag{13}$$

The autoregressive models referring to both wages and unemployment included insignificant components. However, at this stage of the analysis the re-

duction of insignificant components was not carried out. The reduction was carried out only with regard to the initial congruent model.

C. Empirical congruent model

Using the procedure presented in section 4 the congruent model, describing the dependence between the wages and unemployment was obtained. The model reduced to the significant components took the following form:

$$\begin{aligned}
 y_{i,j,t} = & \underset{(0.605992)}{0.25759} + \underset{(0.32561)}{1.55534i} + \underset{(0.275879)}{0.384159j} + \underset{(0.011879)}{0.06988t} \\
 & - \underset{(0.119399)}{0.5518i^2} + \underset{(0.111278)}{0.072165j^2} - \underset{(0.00012)}{0.0004t^2} - \underset{(0.08458)}{0.3897ij} \\
 & - \underset{(0.002265)}{0.00565it} - \underset{(0.002566)}{0.010899jt} + \underset{(0.00964)}{0.02997i^2j} - \underset{(0.0003)}{0.00009i^2t} \\
 & + \underset{(0.010402)}{0.052743ij^2} + \underset{(0.000014)}{0.00006it^2} + \underset{(0.000247)}{0.00017ijt} + \underset{(0.014874)}{0.06462i^3} \\
 & - \underset{(0.015386)}{0.02951j^3} - \underset{(0.000008)}{0.00003t^3} + \underset{(0.000302)}{0.000316j^2t} + \underset{(0.000017)}{0.0001jt^2} \\
 & + \underset{(0.042416)}{0.73533Q_{1t}^*} + \underset{(0.039101)}{0.20754Q_{2t}^*} - \underset{(0.0318657)}{0.02705Q_{3t}^*} - \underset{(0.03269)}{0.3872Q_{4t}^*} \\
 & - \underset{(0.02984)}{0.44331Q_{5t}^*} - \underset{(0.030782)}{0.14771Q_{6t}^*} - \underset{(0.025848)}{0.07501Q_{7t}^*} - \underset{(0.02548)}{0.14575Q_{8t}^*} \\
 & - \underset{(0.02756)}{0.0807Q_{9t}^*} - \underset{(0.038725)}{0.05377Q_{10t}^*} + \underset{(0.031767)}{0.06938Q_{11}^*} + \underset{(0.04451)}{1.0768y_{i,j,t-1}} \\
 & - \underset{(0.04409)}{0.15001y_{i,j,t-2}} + \underset{(0.044276)}{0.19646y_{i,j,t-11}} - \underset{(0.04494)}{0.16087y_{i,j,t-12}} \\
 & - \underset{(0.05018)}{0.17677L^{(1)}y_{i,j,t-1}} + \underset{(0.049885)}{0.14694L^{(1)}y_{i,j,t-2}} - \underset{(0.050076)}{0.21551L^{(1)}y_{i,j,t-11}} \\
 & + \underset{(0.050608)}{0.13605L^{(1)}y_{i,j,t-12}} + \underset{(0.00008)}{0.0002x_{i,j,t-4}} + \underset{(0.000193)}{0.00083L^{(1)}x_{i,j,t-1}} \\
 & - \underset{(0.000199)}{0.00087L^{(1)}x_{i,j,t-10}} + e_{i,j,t}; \tag{14}
 \end{aligned}$$

$$R^2 = 0.996568.$$

The model (14) was obtained from the initial (sufficiently general) congruent model, using the method of *a posteriori* selection. The model contains: trend and seasonality, unemployment rate in the given voivodship and in the neighbouring voivodships with time lags of the 1st, 2nd, 11th and 12th orders (in months), and wages in the given voivodship with time lag of the 4th order. Furthermore, the wages in the neighbouring voivodships with time lags of the 1st and 10th orders have significant influence on the rate of unemployment in the given voivodship. The model (14) does not contain the current wages and salaries.

One should notice, that the current wages and salaries would be present in the unemployment rate model if in the model of the dependence between the

considered processes the trend-seasonal-autoregressive structures had not been taken into account. Then the model would take the following form:

$$y_{i,j,t} = \underset{(0.812304)}{25.846} - \underset{(0.0004745)}{0.004773} x_{i,j,t} + u_{i,j,t} \quad (15)$$

and would be characterized by autodependence in the residuals. The model fit would be very low ($R^2 = 0.0618769$).

Taking into consideration only the trend-seasonal structure of the investigated processes causes, that in the unemployment rate model among the explanatory components the current wages and salaries in the enterprise sector are present. In this case the estimated model is following:

$$\begin{aligned} y_{i,j,t} = & \underset{(1.69264)}{40.9441} - \underset{(1.47611)}{8.51411} i - \underset{(1.48213)}{31.5114} j + \underset{(0.0331213)}{0.491762} t \\ & + \underset{(0.586871)}{5.61938} i^2 + \underset{(0.593534)}{17.3016} j^2 - \underset{(0.000498)}{0.002108} t^2 - \underset{(0.3573)}{5.4682} ij \\ & - \underset{(0.011528)}{0.04511} it - \underset{(0.011531)}{0.09015} jt + \underset{(0.049064)}{0.213544} i^2 j + \underset{(0.00178)}{0.0004} i^2 t \\ & + \underset{(0.045447)}{0.94325} ij^2 + \underset{(0.000064)}{0.00032} it^2 + \underset{(0.001423)}{0.00404} ijt - \underset{(0.07754)}{0.7999} i^3 \\ & - \underset{(0.0779)}{2.6299} j^3 - \underset{(0.000003)}{0.00002} t^3 + \underset{(0.001777)}{0.00143} j^2 t + \underset{(0.00006)}{0.00067} jt^2 \\ & + \underset{(0.165277)}{0.71807} Q_{1t}^* + \underset{(0.165025)}{0.78243} Q_{2t}^* + \underset{(0.163786)}{0.76264} Q_{3t}^* \\ & + \underset{(0.163589)}{0.30425} Q_{4t}^* - \underset{(0.163886)}{0.23988} Q_{5t}^* - \underset{(0.163419)}{0.356201} Q_{6t}^* \\ & - \underset{(0.16339)}{0.35582} Q_{7t}^* - \underset{(0.163451)}{0.44961} Q_{8t}^* - \underset{(0.163583)}{0.505742} Q_{9t}^* \\ & - \underset{(0.163802)}{0.62936} Q_{10t}^* - \underset{(0.164534)}{0.33977} Q_{11t}^* - \underset{(0.0002743)}{0.001723} x_{i,j,t} + u_{i,j,t} \end{aligned} \quad (16)$$

The coefficient R^2 for the model (16) equals 0.833018. The residuals show the spatial and space-time autocorrelation.

In different models the parameters of current wages and salaries differ from one another not only with regard to significance but also the value. Moreover, the parameters of the influence of the wages on the unemployment in various time distances differ from one another with regard to significance, value and sign. This fact should be connected, among other things, with the influence of the wages and salaries on the labour demand – on the one hand – and on the labour activity – on the other hand. It is seemed that these influences can manifest themselves with different strength in different time.

Returning to the model (14) the residual process analysis was done. The model STAR (1, 1) took the following form:

$$\hat{e}_{i,j,t} = -\underset{(0.00798)}{0.00192} + \underset{(0.048389)}{0.02395} e_{i,j,t-1} - \underset{(0.0549243)}{0.059552} L^{(1)} e_{i,j,t-1}. \quad (17)$$

The insignificance of the parameters of the model (17) confirms the lack of the temporal and space-time autocorrelation of the 1st order.

The models STAR of higher orders in time domain were considered as well. However, the parameters in these models were not significant.

6. Conclusion

Taking into account the internal structure of the space-time processes is an important element of modeling of the dependence between these processes. The congruent model, which had the appropriate properties of the residuals, high degree of fit and interpretability of parameters was obtained.

In the presented analysis the simplified assumptions were taken. As regards the autoregressive structure of the investigated processes in the spatial dimension, the attention was limited to the autodependence of the 1st order, which means, that only the so-called „nearest” neighbours were identified. In the models the so-called pure spatial autodependence was not taken into consideration. These questions should be considered in further investigations.

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